

Unparticle in (1+1) dimension with one loop correction

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The possibility of the presence of unparticle has been discussed recently adding a mass like term for gauge field with the Vector Schwinger model at the classical level [8]. A one loop correction due to bosonization is taken into account and investigation is carried over to study the effect of it in connection with the unparticle scenario. It has been found that mass of the gauge boson acquires a generalized expression with the bare coupling constant and the parameters involved in the masslike terms added at the classical level term as well the terms entered in the model during the one loop correction needed for bosonization. So the physical mass viz., unparticle scale gets a new definition. The fermionic propagator is calculated which also agrees with the generalized mass term. A novel restoration of the lost gauge invariance reappears when the ambiguity parameter that entered into the effective action within the one loop correction acquires a specific expression with the bare coupling constant and the parameter involved in the masslike term of the gauge field added at the classical level.

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I. INTRODUCTION

Schwinger model in $(1 + 1)$ dimension [1] has been serving as an interesting field theoretical model since long past. It has been widely studied over the years by several authors in connection with the mass generation of photon, confinement aspect of fermion (quark), charge shielding etc. [2–5]. The description of this model in noncommutative space time is also found to give interesting result [6, 7]. The Schwinger model describes the interaction of massless fermion with the Abelian gauge field. Photon acquires mass via a kind of dynamical symmetry breaking and the fermions disappear from the physical spectra. The possibility of occurrence of unparticle in $(1+1)$ dimension [8] has added a new dimension to this model. The renewed interest in the Schwinger model in connection with the possibility of occurrence of unparticle [8] has thus opened up further scope of investigation on this model. In general unparticle is a scale invariant sector that decouples at large scale and the spectrum of which might be detected in the missing energy momentum distribution [9]. In $(1+1)$ dimension the possibility of occurrence of unparticle has been made possible by adding a masslike term for the gauge field into the action of the Schwinger model at the classical level [8]. It is shown in [8], that it approaches to a free field theory at high energies and a scale invariant theory at low energy.

A one loop correction enters into the effective action of the Schwinger model during bosonization and that contains an ambiguity parameter [10, 11]. The ambiguity parameter reminds us the Jackiw-Rajaraman version of chiral Schwinger model where we have noticed how amazingly ambiguity parameter removed the long suffering of the chiral Schwinger model proposed by Hagen [12]. The Schwinger model with one loop correction needed for bosonization indeed represents a generic description of QED where the usual vector Schwinger model comes as a limiting case from that generic situation [10]. For a specific choice, i.e., for the vanishing value of the ambiguity parameter the model reduces to the usual gauge invariant vector Schwinger model but for the other admissible value of this parameter the phase space structure as well as the physical spectra gets altered remarkably [10]. This generic situation changes the confinement scenario of the fermion too [10]. In fact, the fermion gets liberated as it has been found to be liberated in the Chiral Schwinger model [13–17].

Therefore, it would certainly be an interesting subject of further investigation if one loop correction that enters automatically within the model during bosonization [10] would be taken into account along with the masslike term that has been introduced at the classical level during the description of unparticle [8]. The resulting model in this situation thus would be the Sommerfield (Thirring-Wess) [18, 19] model along with a one loop correction term. To avoid confusion, it should be mentioned that the masslike term for gauge field occurs for different reasons in the model studied in [10, 11] and in the Sommerfield(Thirring-Wess) model [18, 19]. In [10], we have found how the masslike term has got involved as a one loop correction in order to remove the divergence of the fermionic determinant

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during bosonization. However in [19] and [8], the authors have included the masslike term for gauge field at the classical level. In [8], we indeed have observed an exciting extension with the motivation to show the possibility of occurrence of unparticle in (1+1) dimension. Here we are intended to study a combined effect when both the classical masslike term as well as the masslike term entered as one loop correction are present together with the motivation to see whether it can shade any new light in the recently proposed unparticle scenario. Ambiguity parameter involved in the model appeared during the bosonization scheme is exploited here like chiral Schwinger model [13] to show that it has a surprising potential to alter gauge symmetric property of the modified model.

II. STUDY OF PHYSICAL MASS THROUGH CONSTRAINT ANALYSIS

The vector Schwinger model is described by the following generating functional

$$Z[A] = \int d\psi d\bar{\psi} e^{\int d^2x \mathcal{L}_f}, \quad (1)$$

with

$$\mathcal{L}_f = \bar{\psi} \gamma^\mu [i\partial_\mu - e\sqrt{\pi}A_\mu] \psi. \quad (2)$$

In order to get the bosonized lagrangian density from (2) the fermions are to be integrating out one by one [10] and that leads to the following

$$\mathcal{L}_B = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - e \epsilon_{\mu\nu} \partial^\nu \phi A^\mu + \frac{1}{2} a e^2 A_\mu A^\mu. \quad (3)$$

Here a is the regularization ambiguity parameter. Here regularization is done here without maintaining the gauge symmetry of the classical action [10]. If electromagnetic background is now taken into account with masslike term for the gauge field the model reads

$$\mathcal{L}_B = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - e \epsilon_{\mu\nu} \partial^\nu \phi A^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} e^2 (a + \frac{m_0}{e^2}) A_\mu A^\mu. \quad (4)$$

Here Lorentz indices runs over the two values 0 and 1 corresponding to the two space time dimension and the rest of the notations are standard. The antisymmetric tensor is defined with the convention $\epsilon_{01} = +1$. The coupling constant e has one mass dimension in this situation. The parameter m_0 of course has the dimension e^2 which has been introduced in the same way as it has been done initially in the Sommerfield(Thirring-Wess) model [18, 19] and recently which has been studied by Georgi [8] to show the presence of unparticle in (1 + 1) dimension. Let us now proceed to study the phase space structure of the model to observe how physical mass term changes with the inclusion of one loop correction. To this end it is necessary to calculate the the momenta corresponding to the field A_0 , A_1 and ϕ . From the standard definition of the momentum we obtain

$$\pi_0 = 0, \quad (5)$$

$$\pi_1 = F_{01}, \quad (6)$$

$$\pi_\phi = \dot{\phi} - eA_1, \quad (7)$$

where π_0 , π_1 and π_ϕ are the momenta corresponding to the field A_0 , A_1 and ϕ . Using the equations (5), (6) and (7), the canonical hamiltonian density are calculated:

$$\mathcal{H}_c = \frac{1}{2} (\pi_\phi + eA_1)^2 + \frac{1}{2} \pi_1^2 + \frac{1}{2} \phi'^2 + \pi_1 A'_0 - eA_0 \phi' - \frac{1}{2} e^2 (a + \frac{m_0}{e^2}) (A_0^2 - A_1^2). \quad (8)$$

Note that $\omega = \pi_0 \approx 0$, is the familiar primary constraint of the theory. The preservation of the constraint $\omega(x)$ requires $[\omega(x), H(y)] = 0$, which leads to the Gauss' law as a secondary constraint:

$$\tilde{\omega} = \pi'_1 + e\phi' + e^2 (a + \frac{m_0}{e^2}) A_1 \approx 0. \quad (9)$$

The constraints (5) and (9) form a second class set. Treating (5) and (9) as strong condition one can eliminate A_0 and obtain the reduced hamiltonian density as follows.

$$\mathcal{H}_r = \frac{1}{2}(\pi_\phi + eA_1)^2 + \frac{1}{2m_0}(\pi'_1 + e\phi')^2 + \frac{1}{2}(\pi_1^2 + \phi'^2)^2 + \frac{1}{2}e^2(a + \frac{m_0}{e^2})A_1^2. \quad (10)$$

According to the Dirac's prescription of quantizing [20] the Poisson brackets become inadequate for a theory possessing second class constraint in its space. This type of system however remains consistent with the Dirac brackets [20]. It is straightforward to show that the Dirac brackets between the fields describing the reduced hamiltonian (10) remain canonical. Using the canonical Dirac brackets the following first order equations of motion are found out from the reduced Hamiltonian density (10).

$$\dot{A}_1 = \pi_1 - \frac{1}{e^2(a + \frac{m_0}{e^2})}(\pi''_1 + e\phi''), \quad (11)$$

$$\dot{\phi} = \pi_\phi + eA_1, \quad (12)$$

$$\dot{\pi}_\phi = (1 + \frac{1}{(a + \frac{m_0}{e^2})})\phi'' + \frac{1}{e(a + \frac{m_0}{e^2})}\pi''_1, \quad (13)$$

$$\dot{\pi}_1 = -e\pi_\phi - e^2(a + \frac{m_0}{e^2} + 1)A_1. \quad (14)$$

A little algebra converts the above first order equations (11), (12), (13) and (14) into the following two second order differential equations:

$$[\square + e^2(a + \frac{m_0}{e^2} + 1)]\pi_1 = 0, \quad (15)$$

$$\square[\pi_1 + e(a + \frac{m_0}{e^2} + 1)\phi] = 0. \quad (16)$$

Equation (15) describes a massive boson field with square of the mass $m = e^2(a + \frac{m_0}{e^2} + 1)$ whereas equation (16) describes a massless scalar field. The equation (15) clearly shows that the physical mass acquires a generalized expression with the parameters involved in the masslike term at the classical level as well as the parameter appeared due to the one loop correction entered during bosonization. The change that entered into the physical mass term has certainly taken place due to the very one loop correction and consequently the definition of unparticle scale [8] also has acquired a new expression with an adjustable parameter a . The nature of the theoretical spectrum becomes more transparent if we calculate the fermionic propagator to which we will now turn.

III. CALCULATION OF FERMIONIC PROPAGATOR

To calculate fermion propagator one needs to work with the original fermionic model. The calculation of fermionic propagator is analogous to the Chiral Schwinger model [15, 16] and the so called nonconfining Schwinger model [10]. The effective action obtained by integrating out ϕ from the bosonized action (4) is

$$S_{eff} = \int d^2x \frac{1}{2}[A_\mu(x)M^{\mu\nu}A_\nu(x)], \quad (17)$$

where,

$$M^{\mu\nu} = e^2(a + \frac{m_0}{e^2})g^{\mu\nu} - \frac{\square + e^2}{\square}\tilde{\partial}^\mu\tilde{\partial}^\nu. \quad (18)$$

Here we have used the standard notation $\tilde{\partial}^\mu = \epsilon^{\mu\nu}\partial_\nu$. The gauge field propagator is just the inverse of $M^{\mu\nu}$ and it is found to be

$$\Delta_{\mu\nu}(x - y) = \frac{1}{e^2(a + \frac{m_0}{e^2})}[g_{\mu\nu} + \frac{\square + e^2}{\square(\square + e^2(a + \frac{m_0}{e^2} + 1))}\tilde{\partial}_\mu\tilde{\partial}_\nu]\delta(x - y). \quad (19)$$

Note that the two poles of propagator are found at the expected positions. One is at zero and another is at $e^2(a + \frac{m_0}{e^2} + 1)$ indicating a massless and a massive excitations respectively.

We are now in a stage to calculate the Green function of the Dirac operator. To do that let us consider the following Ansatz for the Green function of the Dirac operator $(i\partial - eA)$.

$$G(x, y; A) = e^{ie(\Phi(x) - \Phi(y))} S_F(x - y), \quad (20)$$

where S_F is the free, massless fermion propagator and Φ will be determined when the Ansatz (20) will be plugged into the equation for the Green function. It will enable us to construct the propagator of the original fermion ψ . From the standard construction the Green function is found out as

$$G(x, y; A) = e^{ie \int d^2z A^\mu(z) J_\mu(z)} S_F(x - y), \quad (21)$$

where the fermionic *current* J_μ is given by the following expression.

$$J_\mu = (\partial_\mu^z + \gamma_5 \tilde{\partial}_\mu^z)(D_F(z - x) - D_F(z - y)). \quad (22)$$

Here D_F represents the propagator of a massless free scalar field. In order to avoid singularity such propagators have to be infra-red regularized in $(1 + 1)$ dimensions [2].

$$D_F(x) = -\frac{i}{4\pi} \ln(-\mu^2 x^2 + i0). \quad (23)$$

Here μ stands for the infra-red regulator mass.

Finally we obtain the fermion propagator by functionally integrating $G(x, y; A)$ over the gauge field:

$$\begin{aligned} S'_F &= \int \mathcal{D}A e^{\frac{i}{2} \int d^2z (A_\mu(z) M^{\mu\nu} A_\nu(z) + 2e A_\mu J^\mu)} S_F(x - y) \\ &= \mathcal{N} \exp\left[\frac{D_F}{(a + \frac{m_0}{e^2})(a + \frac{m_0}{e^2} + 1)} + \frac{\Delta_F(m^2 = e^2(a + \frac{m_0}{e^2} + 1))}{a + \frac{m_0}{e^2} + 1}\right] S_F. \end{aligned} \quad (24)$$

Here Δ_F is the propagator of a massive free scalar field and \mathcal{N} is a wave function renormalization factor.

The theoretical spectrum (15) and (16) as well as the fermionic propagator (24) therefore, make the fact confirm that the mass acquires a generalized expression with bare coupling constant and the parameters involved within the masslike terms for the gauge field. Thus a new definition of unparticle scale emerges out with the introduction of one loop correction holding the hands of generalized physical mass term appeared in (15). The emerging out of this new definition of unparticle scale is an important as well as interesting aspect of taking the one loop correction into account and it will certainly be of important use in the area of physics where unparticle is expected to show a prominent role. It is known that in the vector Schwinger model the mass that generates via dynamical symmetry breaking depend only on the bare coupling constant and it does not contain any such parameters like m_0 or a . In fact, in the bosonized vector Schwinger model no such parameter appears when bosonization is done maintaining the gauge symmetry. It is true that setting $a = 0$, one can obtain vector Schwinger model [1] from the bosonized lagrangian of the so called nonconfining Schwinger model [10] where bosonization is done without maintaining the gauge symmetry. But this trivial setting of $a = 0$ does not work to bring back the gauge symmetry in this generalized situation where masslike terms from both the classical as well as one loop corrected level are present. However there is some surprise which we are going to uncover.

IV. RESTORATION OF GAUGE SYMMETRY EXPLOITING THE AMBIGUITY PARAMETER

When a model contains an ambiguity parameter we have seen several times to exploit it to get some thing new and interesting or to use it as a remedial measure to rescue it from some unphysical situation through which it is suffering without violating any physical principle [13–17, 21–24]. To get back the gauge symmetry of this modified model we would like to exploit the arbitrariness of the ambiguity parameter once more. Without violating any physical principal the said arbitrariness allows us to set

$$(a + \frac{m_0}{e^2}) = 0. \quad (25)$$

A novel thing it renders is that it gives an expression (definition) of a in terms of the parameters involved in the theory and simultaneously it brings a remarkable change in the constraint structure of the theory. In fact, this definition of a

represents a singularity in the phase space structure of the theory where the constraint structure gets converted into a first set from the existing second class structure and the following two first class constraints result.

$$\pi_0 \approx 0, \quad (26)$$

$$\pi'_1 + e\phi' \approx 0. \quad (27)$$

So with this particular expression of the ambiguity parameter a the system is now having the above two first class constraint (26) and (27) and to single out the physical degrees of freedom we now need two gauge fixing conditions. The following two conditions would be the appropriate gauge fixing at this stage.

$$A_0 \approx 0 \quad (28)$$

$$A'_1 \approx 0 \quad (29)$$

When the gauge fixing conditions (28) and (29) are plugged strongly into the hamiltonian (8) along with the two first class constraints (26) and (27) the hamiltonian (8) acquires the following simplified form.

$$H_{RF} = \int dx \left[\frac{1}{2} \pi_\phi^2 + \frac{1}{2} e^2 \phi^2 + \frac{1}{2} \phi'^2 \right] \quad (30)$$

The Dirac brackets [20] for the fields describing the reduced hamiltonian (30) are found to remain canonical. The hamiltonian (30) leads to the following equations of motion when the canonical Dirac brackets are used for computation of equations of motion.

$$\dot{\phi} = \pi_\phi \quad (31)$$

$$\dot{\pi}_\phi = \phi'' - e^2 \phi \quad (32)$$

The above two equations give a single Klein-Gordon type second order differential equation

$$[\square + e^2]\phi = 0 \quad (33)$$

It describes a massive boson with square of the mass e^2 . Note that the mass does not contain any of the parameter a or m_0 . A careful look indicate that it is nothing but the reappearance of the usual vector Schwinger model just by exploiting the ambiguity parameter a when the classical masslike term for the gauge field is also present in the model. In fact, with the expression of a standing in equation (25) the modified model maps onto the usual vector Schwinger model. The trivial choices $a = 0$ and $m_0 = 0$, of course, convert this modified model to the usual Vector Schwinger model. However the above choices have no concrete basis like the situation that is allowed from the exploitation of the arbitrariness involved in the theory.

Needless to mention that vector Schwinger model is known to possess gauge symmetry. The presence of two first class constraint in the phase space in this situation also indicates that the model with this particular definition of a has got back the gauge symmetry in its usual phase space where the first class constraints will work as the generator of the gauge transformation. There are different ways to restore gauge invariance of a gauge nonsymmetric model. Mitra-Rajaraman's prescription [25, 26] is a good example of that where restoration of gauge symmetry of a given model is possible in its usual phase space. Some other prescriptions are also available where the gauge symmetry restoration takes place in the extended phase space [27–30]. However the restoration of gauge symmetry through a specific non vanishing expression of the ambiguity parameter a is a novel thing which may have some greater importance elsewhere especially in the physics at high energy regime where unparticle is expected to have a very important role.

V. CONCLUSION

So what we have achieved here is that the physical mass acquires a generalized expression with the parameter involved in the masslike terms at the classical level as well as the masslike terms entered into the model due to one loop correction. As the physical mass acquires a new generalized expression a new definition in the unparticle scale automatically emerges out which contains an adjustable parameter a . It might have interesting use in regime

of physics where unparticle is expected to play a prominent role. The second surprises what we have achieved is the obtaining of an expression of the ambiguity parameter a with the exploitation of the arbitrariness which entered automatically into the theory during bosonization. This definition of a brings back the symmetry of the effective action in the usual phase space though a classical masslike term for gauge fields breaks the symmetry of the fermionic action to start with.

As a concluding remark we would like to mention that the term $\frac{1}{2}e^2(a + \frac{m_0}{e^2})A^\mu A_\mu$ though looks like a gauge fixing term in reality it is not the case. With this term the gauge redundancy of the vector Schwinger model though gets fixed up and it turns into a gauge noninvariant model nevertheless it can not be thought of as gauge fixing because introduction of this term brings a drastic change in the spectrum of the model. The mass of the massive boson acquires a generalized expression. A massless boson also appears in the spectrum. Instead of the term $\frac{1}{2}e^2(a + \frac{m_0}{e^2})A^\mu A_\mu$ if the term $\frac{\alpha}{2e^2}(\partial_\mu A^\mu)^2$ is inserted into the starting lagrangian of the vector Schwinger model it works as a true gauge fixing term since in that situation one gets a parameter free mass term for the photon in spite of the presence of the parameter α in the starting lagrangian and no extra massless boson dose appear there. One more thing we would like to mention here is that the exploitation of the ambiguity parameter at our will to bring back the symmetry of the model dose not violate any physical principle. The ambiguity in the regularization has been exploited by different authors in different times in (1+1) dimensional QED and Chiral QED for differen reasons and different rescues have been resulted in [13–17, 21–24]. The most remarkable one is the chiral Schwinger model studied by Jackiw and Rajaraman [13] where they saved the long suffering of the chiral generation of the Schwinger model due to Hagen [12] from the non-unitary problem.

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